Enrollment No: _____ Exam Seat No: _____

C. U. SHAH UNIVERSITY
Winter Examination-2020

Subject Name: Operations Research

Subject Code: 5SC01OPR1 **Branch: M.Sc. (Mathematics)**

Semester: 1 **Date: 15/03/2021** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the following questions.** **(07)**

a. How degeneracy is recognized when using the simplex algorithm? **02**

b. Define: (i) Basic solution (ii) Basic feasible solution. **02**

c. If dual has an unbounded solution, then primal has _____. **01**

d. Define: Convex Set. **01**

e. The graphical method can only be used when there are_____ decision variables. **01**

Q-2 **Attempt all questions.** **(14)**

a. A farmer has a 100-acre farm. He can sell all tomatoes, lettuce or radishes and can get a price of Rs.1.00 per kg for tomatoes, Rs.0.75 a heap for lettuce and Rs.2.00 per kg for radishes. The average yield per acre is 2,000 kg of tomatoes, 3,000 heaps of lettuce and 1,000 kg of radishes. Fertilizers are available at Rs.0.50 per kg and the amount required per acre is 100 kg each for tomatoes and lettuce and 50 kg for radishes.Labour required for sowing, cultivating and harvesting per acre is 5 man-days for tomatoes and radishes and 6 man-days for lettuce. A total of 400 man-days of labour are available at Rs. 20 per man-day. Formulate this problem as a linear programming model to maximize the farmer's total profit. **07**

b. Solve the following Linear Programming Problem by using Penalty (Big-M) method. **07**

Maximize $z = 2x_1 + x_2 + 3x_3$
Subject to

$$x_1 + x_2 + 2x_3 \leq 5$$
$$2x_1 + 3x_2 + 4x_3 = 12$$

and $x_1, x_2, x_3 \geq 0$

OR

Q-2 **Attempt all questions.** **(14)**

a. Using graphical method to solve the following LP problem. **07**

Maximize $z = 6x_1 + 4x_2$

Subject to

$$-2x_1 + x_2 \leq 2$$
$$x_1 - x_2 \leq 2$$
$$3x_1 + 2x_2 \leq 9$$

and $x_1, x_2 \geq 0$

b. Solve the followingLPproblem by using simplex method. **07**

Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subject to

$$x_1 + 2x_2 + x_3 \leq 430$$
$$3x_1 + 2x_3 \leq 260$$
$$x_1 + 4x_2 \leq 420$$

and $x_1, x_2, x_3 \geq 0$

Q-3 **Attempt all questions.** **(14)**

a. Write the dual of the following linear programming problem. **07**

(i) Minimize$z_x = x_1 - 2x_2 + 3x_3$

Subject to

$$-2x_1 + x_2 + 3x_3 = 2$$
$$2x_1 + 3x_2 + 4x_3 = 1$$

and $x_1, x_2 \geq 0$

(ii) Maximize$z_x = x_1 + 2x_2 + x_3$

Subject to

$$2x_1 + x_2 - x_3 \leq 2$$
$$-2x_1 + x_2 - 5x_3 \geq -6$$
$$4x_1 + x_2 + x_3 \leq 6$$

and $x_1, x_2, x_3 \geq 0$

b. Use two-phase method to solve the following LP problem. **07**

Minimize $z = 5x_1 + 8x_2$

Subject to

$$3x_1 + 2x_2 \geq 3$$
$$x_1 + 4x_2 \geq 4$$
$$x_1 + x_2 \leq 5$$

and $x_1, x_2 \geq 0$

OR

Q-3 **Attempt all questions.** **(14)**

a. Use graphical method to solve the following LP problem. **05**

Maximize $z = 3x_1 + 2x_2$

Subject to

$$x_1 - x_2 \geq 1$$
$$x_1 + x_2 \geq 3$$

and $x_1, x_2 \geq 0$

b. Prove that dual of dual is primal. **05**

c. If $S = \{(x_1, x_2): x_1 \cdot x_2 \geq 1, x_1, x_2 \geq 0\}$, then show that S is convex set. **04**

SECTION – II

Q-4 **Attempt the following questions.** **(07)**

a. Explain general mathematical model of transportation problem. **02**

b. Write Hessian matrix with four decision variables. **02**

c. Which method gives best approximation to find out initial basic feasible solution of transportation problem? **01**

d. True/False: The Assignment problem is a special type of linear programming problem. **01**

e. True/False: A dummy row or column is introduced in the transportation method in order to handle an unbalanced problem. **01**

Q-5 **Attempt all questions.** **(14)**

a. Explain Least cost method. Find the initial basic feasible solution to the following transportation problem using North-west corner method. **07**

Origin	Destination				Supply
	D_1	D_2	D_3	D_4	
O_1	06	04	01	05	14
O_2	08	09	02	07	16
O_3	04	03	06	02	05
Demand	06	10	15	04	35

b. There are four jobs to be assigned to five machines. Only one job can be assigned to one machine. The amount of time in hours required for the jobs per machine are given in the following table **07**

Jobs	Machines				
	A	B	C	D	E
1	04	03	06	02	07
2	10	12	11	14	16
3	04	03	02	01	05
4	08	07	06	09	06

Find an optimum assignment of job to machines to minimize the total processing time and also find out for which machine no job is assigned. What is the total processing time to compete the job?

OR

Q-5 **Attempt all questions.** **(14)**

a. Solve the following transportation problem by using MODI method. **07**

Source	Destination				Supply
	A	B	C	D	
1	11	20	07	08	50
2	21	16	20	12	40
3	08	12	08	09	70
Demand	30	25	35	40	

b. Explain Hungarian method for solving assignment problem. **07**

Q-6 **Attempt all questions.** **(14)**

a. Solve the given linear programming problem using dual simplex method. **07**

Minimize $z = 3x_1 + x_2$

Subject to

$$x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 2$$

and $x_1, x_2 \geq 0$

b. Use the method of Langrage's multipliers to solve the following non-linear programming problem. **07**

Minimize $z = x_1^2 + x_2^2 + x_3^2$

Subject to

$$x_1 + x_2 + 3x_3 = 2$$
$$5x_1 + 2x_2 + x_3 = 5$$

and $x_1, x_2, x_3 \geq 0$

OR

Q-6 **Attempt all questions.** **(14)**

a. Explain the steps of Modified distributive method for transportation problem. **07**

b. Use the Kuhn-Tucker conditions to solve the following non-linear programming problem. **07**

Maximize $z = 2x_1 - x_1^2 + x_2$

Subject to

$$2x_1 + 3x_2 \leq 6$$
$$2x_1 + x_2 \leq 4$$

and $x_1, x_2 \geq 0$